A NOTE ON THE MEMORY THEORY OF CREEP IN METALS

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In their modern form memory theories of creep are to a significant extent constructed on the basis of formal considerations, in particular, the principle of superposition, according to which the effects of the stresses and temperature acting at a given moment ξ are not disturbed by stresses applied at other moments of time or temperature changes occurring at other moments of time. That the application of the memory theory to metals nevertheless leads to satisfactory results in a number of important cases is demonstrated below.

Beginning with Volterra, various generalizations of the Boltzmann-Volterra memory theory have been proposed for the computation of the nonlinear relation between creep and stress [1]. The starting equation of the nonlinear theory may be written in the form

$$\varepsilon_{ij}(t) = e_{ij}(t) + p_{ij}(t),$$

$$p_{ij}(t) = \int_0^t Q_{ij}\left[t-\xi,\ \sigma_{\alpha\beta},(\xi)\,T\ (\xi)\right]d\xi \qquad (1)$$

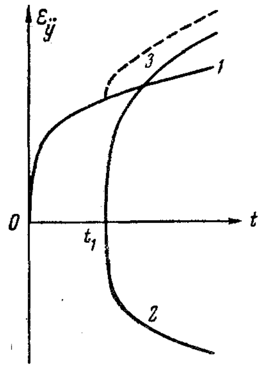

Fig. 1.

where e_{ij} is the elastic deformation, linked with the stress by Hooke's Law, supplemented by temperature terms; p_{ij} is the creep, and T is the temperature.

It will be assumed that the form of Q_{ij} in equation (1) is such that the theoretical and experimental curves coincide for invariable stresses and temperature.

Let the values $\sigma_{ij} = \sigma_{ij}^{(1)}$ and $T = T_1$, which are invariable when $t < t_1$, assume the values $\sigma_{ij} = \sigma_{ij}^{(2)}$ and $T = T_2$ at the moment $t = t_1$. It follows from equation (1) that

$$\varepsilon_{ij}(t) = e_{ij}(t) + \int_0^t Q_{ij}(t-\xi,\sigma_{\alpha\beta}^{(1)},T_1)\,d\xi \qquad (t \leqslant t_1)$$

$$\varepsilon_{ij}(t) = e_{ij}(t) + \int_0^{t_1} Q_{ij}(t-\xi,\sigma_{\alpha\beta}^{(1)},T_1)\,d\xi + \int_{t_1}^t Q_{ij}(t-\xi,\ \sigma_{\alpha\beta}^{(2)},T_2)\,d\xi \qquad (t > t_1).$$

If on the right side of the latter equation, we first add and then subtract the term

$$\int_{t_1}^t Q_{ij}(t-\xi,\sigma_{\alpha\beta}^{(1)},T_1)\,d\xi\ ,$$

we get

$$\varepsilon_{ij}(t) = e_{ij}(t) + \int_0^t Q_{ij}(t-\xi,\sigma_{\alpha\beta}^{(1)},T_1)\,d\xi - \int_{t_1}^t Q_{ij}(t-\xi,\sigma_{\alpha\beta}^{(1)},T_1)\,d\xi + $$
$$+ \int_{t_1}^t Q_{ij}(t-\xi,\sigma_{\alpha\beta}^{(2)},T_2)\,d\xi \qquad (t > t_1)\ .$$

Hence in order to obtain the theoretical curves for the complete process with $t > t_1$ (broken line in Fig. 1), it is necessary to add the ordinates of the curves shown by a solid line in Fig. 1 for each of the six planes ε_{ij}, t:

$$1 - (\sigma_{\alpha\beta} = \sigma_{\alpha\beta}^{(1)},\ T = T_1),\quad 2 - (\sigma_{\alpha\beta} = \sigma_{\alpha\beta}^{(1)},\ \ T = T_1),\quad 3 - (\sigma_{\alpha\beta} = \sigma_{\alpha\beta}^{(2)},\ \ T = T_2)\ .$$

The same constructions are also possible for repeated stepwise changes in stresses and temperature, if we have creep curves for invariable σ_{ij} and T, equal to those acting at every step. Theoretical curves in the coordinates (p_{ij}, t) may be constructed analogously. In fact, if a thermal plastic deformation enters into e_{ij}, in addition to elastic deformation, the construction should be based precisely on the coordinates (p_{ij}, t).

This graphic method generalizes the method proposed by Leaderman [2] for the construction of a theoretical curve –

for the one-dimensional process, in which a stress is periodically applied to the specimen and then removed.

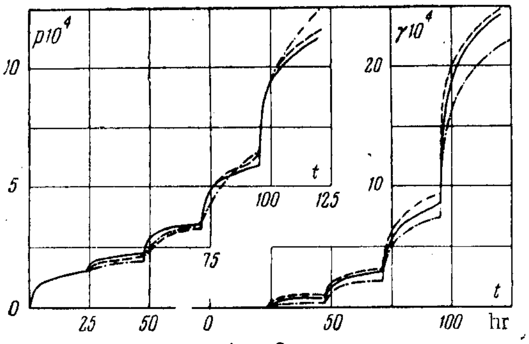

Fig. 2.

A comparison of the memory theory with experimental data [3-6] on pure copper, carbon steel, and duralumin in simple tension for stepwise variation in stress or temperature disclosed that, in the case of decreasing stress or temperature, the theory shows much greater recovery than that observed experimentally. In the case of increasing stress or temperature, the memory theory leads to the same satisfactory results as the theory applied in [7]. The application of the Nadai-Davis strain-hardening theory gives worse results for increasing stress.

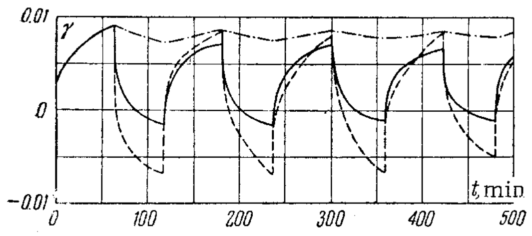

Fig. 3.

The memory theory is in good agreement with the experiments of Johnson, Henderson and Mathur [8] for thin-walled, tubular specimens of carbon steel and aluminum and magnesium alloys. The specimens were loaded at a fixed temperature with a steady axial tensile stress and a stepwise increasing torque. By way of example, Fig. 2 shows the experimental curves for the creep of magnesium alloy at 20°C (solid lines), the curves for the memory theory (broken lines), and the curves for the theory used in [7] (dot-dash lines) in the coordinate systems (p,t) and (γ, t) (p is the axial creep and γ the shear creep). The axial stress was equal to 472 kg/cm^2, while the tangential stress increased in steps of 94.5 kg/cm^2 in each 24-hour period. The fact that the memory theory is in better agreement with the above results than the theory used in [7] is attributable to the fact that the former qualitatively, and correctly, takes into account the deformation anisotropy due to combined loading, which the other theory [7] does not.

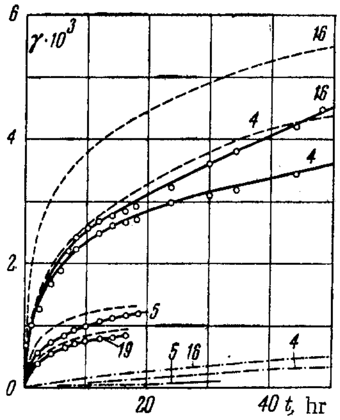

Fig. 4.

Considerable deformation anisotropy develops when the stress changes sign. Thus, in the case of alternate forward and reverse torsion of thin-walled tubular specimens, the directions of the principal stresses change instantaneously by 90°. In such experiments softening rather than the expected strain hardening is observed when the tangential stress τ changes sign [9]. This effect is also considered by the memory theory. The results of experiments [9, 10] on carbon steel and duralumin (solid lines) and curves based on the memory theory (broken lines) are shown in Figs. 3 and 4, respectively. The shear creep is denoted by γ. The carbon steel was tested at 500°C and $\tau = \pm 12.8$ kg/mm^2. Alloy specimens 19, 5, 4, and 16 (D16T duralumin) were tested at 150°C and τ = 10.1, 11.56, 14.00, and 14.74 kg/mm^2, respectively. The moment the stress changed sign was taken as the origin of the time readings. The strain accumulated up to this point was taken as the new origin of the deformation readings. The strain accumulated up to the time of the first measurement is not considered. The theory of [7] (dot-dash line) describes the results of the experiments much less satisfactorily than the memory theory.

The following conclusions with respect to the memory theory and the theory used in [7] can be made on the basis of the experiments discussed. In the case of simple loading with increasing stress or temperature the application of the memory theory and the theory of [7] leads to almost equally satisfactory results. With decreasing stress or temperature, the theory of [7] gives better results. In the case of combined loading with non-decreasing stresses, the memory theory gives better results.

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